Problems on Probability Distributions & Random variable

1. For the following probability distribution, Find Mean, variance and standard Deviation.

$X = x_i$	1	2	3	4
$P(X=x_i)$	0.1	0.2	0.3	0.4

$\Sigma x_i p_i$

Sol: Mean or expected value: $\mu = \Sigma x_i p_i$

$\Sigma x_i p_i$

$$\mu = (1)(0.1) + (2)(0.2) + (3)(0.3) + 4(0.4)$$

$$= 0.1 + 0.4 + 0.9 + 1.6$$

$$\mu = 3$$

Variance $\sigma^2 = \Sigma x_i^2 p_i - \mu^2$

$$\sigma^2 = (1)^2(0.1) + (2)^2(0.2) + (3)^2(0.3) + (4)^2(0.4) - (3)^2$$

$$= 1(0.1) + 4(0.2) + 9(0.3) + 16(0.4) - 9$$

$$= 0.1 + 0.8 + 2.7 + 6.4 - 9$$

$$\sigma^2 = 10 - 9 = 1.$$

$$S.D = \sigma = \sqrt{\sigma^2} = +\sqrt{1} = 1$$

2. A random variable x has the following probability distribution. Find its mean and S.D

$X = x_i$	0	1	2	3
$P(X=x_i)$	$1/3$	$1/2$	0	$1/6$

Sol: Mean $= \mu = \sum x_i p_i$

$$\mu = 0(1/3) + 1(1/2) + 2(0) + 3(1/6)$$

$$= 0 + \frac{1}{2} + 0 + \frac{1}{2}$$

$$\mu = 1$$

Variance $\sigma^2 = \sum x_i^2 p_i - \mu^2$

$$\sigma^2 = 0^2(1/3) + 1^2(1/2) + (2)^2(0) + (3)^2(1/6) - 1^2$$

$$= 0(1/3) + 1(1/2) + 4(0) + 9(1/6) - 1$$

$$= 0 + 1/2 + 0 + \frac{3}{2} - 1$$

$$\sigma^2 = 2 - 1 = 1$$

$$S.D = +\sqrt{\sigma^2} = \sqrt{1} = 1$$

(3) When a cubical die is thrown find its mean and S.D

Sol: When a cubical die is thrown, the sample space $S = \{1, 2, 3, 4, 5, 6\}$. Define Random Variable X = number on its face

ie $X = \{1, 2, 3, 4, 5, 6\}$

$X = x_i$	1	2	3	4	5	6
$P(X = x_i)$	1/6	1/6	1/6	1/6	1/6	1/6

Sol: Mean $= \mu = \sum x_i p_i$

$$\mu = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(\tfrac{1}{6}) + 6(\tfrac{1}{6})$$

$$= \frac{1}{6}[1+2+3+4+5+6] = \frac{21}{6} = \frac{7}{2}$$

Variance $\sigma^2 = \Sigma x_i^2 p_i - \mu^2$

$$\sigma^2 = 1^2(1/6) + 2^2(1/6) + 3^2(1/6) + 4^2(1/6) + 5^2(1/6) + 6^2(1/6) - \left(\frac{7}{2}\right)^2$$

$$= 1(1/6) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6) - \frac{49}{4}$$

$$= \frac{1}{6}\left[1+4+9+16+25+36\right] - \frac{49}{4}$$

$$= \frac{91}{6} - \frac{49}{4}$$

$$= 15.17 - 12.25$$

$$\sigma^2 = 2.92$$

$$S.D = +\sqrt{\sigma^2} = \sqrt{2.92} = 1.709$$

(4) *** (DECEMBER 2007)

Construct a probability distribution based on the following frequency distributions.

Outcome :	2	4	6	8	10	12	15
Frequency :	24	22	16	12	7	3	1

Compute the expected value of the outcome

Sol:

Sum of the frequencies $N = \Sigma f =$

$24 + 22 + 16 + 12 + 7 + 3 + 1 = 85$

Convert the given frequency distribution into probability distribution

outcome	frequency	Probability
2	24	$24/85 = 0.282$
4	22	$22/85 = 0.259$
6	16	$16/85 = 0.188$
8	12	$12/85 = 0.141$
10	7	$7/85 = 0.082$
12	3	$3/85 = 0.035$
15	1	$1/85 = 0.012$

$N = \Sigma f = 85$ $\quad \Sigma p_i = 1$

$X = x_i$	2	4	6	8	10	12	15
$P(X = x_i)$	0.28	0.26	0.19	0.14	0.08	0.04	0.01

$\Sigma p_i = 1$

Expected value $\mu = \Sigma x_i p_i$

$$\mu = (2)(0.28) + 4(0.26) + 6(0.19) + 8(0.14) + 10(0.08) + 12(0.04) + 15(0.01)$$

$$= 0.56 + 1.04 + 1.14 + 1.12 + 0.8 + 0.48 + 0.150$$

$$\mu = 5.29$$

Convert the given frequency distribution into probability distribution

outcome	frequency	Probability
2	24	$24/85 = 0.282$
4	22	$22/85 = 0.259$
6	16	$16/85 = 0.188$
8	12	$12/85 = 0.141$
10	7	$7/85 = 0.082$
12	3	$3/85 = 0.035$
15	1	$1/85 = 0.012$
	$N = \Sigma f = 85$	$\Sigma p_i = 1$

$X = x_i$	2	4	6	8	10	12	15
$P(X = x_i)$	0.28	0.26	0.19	0.14	0.08	0.04	0.01

$\Sigma p_i = 1$

Expected value $\mu = \Sigma x_i p_i$

$$\mu = (2)(0.28) + 4(0.26) + 6(0.19) + 8(0.14) + 10(0.08) + 12(0.04) + 15(0.01)$$

$$= 0.56 + 1.04 + 1.14 + 1.12 + 0.8 + 0.48 + 0.15$$

$$\mu = 5.29$$

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Problems on Binomial Distribution

① In a family of 5 children, find the probability of having

(a) no male child

(b) one male child

(c) two male children

(d) Three male children

(e) Four male children

(f) Five male children

(g) All female children

(h) atleast one male child

(i) more than two male children

(j) more than 4 male children

(k) Less than 3 male children

(l) Less than or equal to 2 male children

(m) atleast 2 male children

(n) at least 4 male children

(o) atmost 3 male children

(p) more than or equal to 3 male children

(q) 2 female children

n. H.T $\frac{1}{2}$ independent or dependent

Sol: Given a family with five children

$n = 5$ prob of male child $= p = \frac{1}{2}$

prob of female child $= q = 1-p$

$= 1 - \frac{1}{2} = \frac{1}{2}$

(a) Prob of no male child $(r=0)$

$P(X=r) = {}^nC_r \, p^r \, q^{n-r}$

$P(X=0) = {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0}$

$= 1 \cdot \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

(b) Prob of one male child $P(X=1)$

$r = 1$

$P(X=1) = {}^5C_1 \, p^1 \, (q)^{n-1}$

$= {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$

$= 5 \cdot \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$

$= 5 \left(\frac{1}{2}\right)^5 = 5/32$

$\frac{3+3+1}{7}$

(c) Prob of two male children

calculator

$r = 2$ $P(X=2) = ?$

$\boxed{5 \wedge 10}$

$P(X=2) = {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2}$

$= 10 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$

$= 10 \cdot \left(\frac{1}{2}\right)^5$

$= 10 \cdot \left(\frac{1}{32}\right) = \frac{10}{32}$

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(d) Prob of Three male children

$r=3$ $P(X=3)=?$

$$P(X=r) = {}^{n}C_{r}\,(p)^{r}\,(q)^{n-r}$$

$$P(X=3) = {}^{5}C_{3}\left(\tfrac{1}{2}\right)^{3}\left(\tfrac{1}{2}\right)^{5-3}$$

$$= 10\cdot\left(\tfrac{1}{2}\right)^{3}\left(\tfrac{1}{2}\right)^{2}$$

$$= 10\cdot\left(\tfrac{1}{2}\right)^{5}$$

$$P(X=3) = \frac{10}{32}$$

${}^{5}C_{3}$

$C(5,3) = \frac{5!}{2!\,3!}$

(e) Prob of Four male children

$r=4$ $P(X=4)=?$

$$P(X=4) = {}^{5}C_{4}\left(\tfrac{1}{2}\right)^{4}\left(\tfrac{1}{2}\right)^{5-4}$$

$$= 5\cdot\left(\tfrac{1}{2}\right)^{4}\left(\tfrac{1}{2}\right)^{1}$$

$$= 5\cdot\left(\tfrac{1}{2}\right)^{5} = 5\cdot\left(\tfrac{1}{32}\right)$$

$$P(X=4) = \frac{5}{32}$$

(f) Prob of 5 male children

$r=5$ $P(X=5)=?$

$$P(X=5) = {}^{5}C_{5}\left(\tfrac{1}{2}\right)^{5}\left(\tfrac{1}{2}\right)^{5-5}$$

$$= 1\cdot\left(\tfrac{1}{2}\right)^{5}\left(\tfrac{1}{2}\right)^{0}$$

$$= 1\cdot\left(\tfrac{1}{2}\right)^{5}$$

$$P(X=5) = \frac{1}{32}$$

$^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 =$

(g) Prob of all female children

ie $\Rightarrow$ no male children $\Rightarrow r=0$

$P(X=0) = {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0}$

$= 1 \cdot 1 \cdot \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$P(X=0) = 1/32$

$x=r$	0	1	2	3	4	5
$P(X=r)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

(h) Prob of atleast one male child

$P(X \geq 1)$

$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32}$

$P(X \geq 1) = \frac{31}{32}$

**

OR $\quad P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - 1/32$

$= \frac{32-1}{32} = \frac{31}{32}$

$P(X \geq 1) = \frac{31}{32}$

$P(X \geq 1) = 1 - P(X=0)$

(i) more than or equ 2 male children

$$P(x \geq 2) = ?$$

$$P(x>2) = 1 - P(x<2)$$

$$= 1 - \{P(x=0) + P(x=1)\}$$

$$= 1 - \left\{\frac{1}{32} + \frac{5}{32}\right\}$$

$$= 1 - \frac{6}{32}$$

$$P(x>2) = \frac{26}{32}$$

(j) more than 4 male children

$$P(x>4) = ?$$

$$P(x>4) = P(X=5)$$

$$= \frac{1}{32}$$

(k) Less than 3 male children

$$P(x<3) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{1}{32} + \frac{5}{32} + \frac{10}{32}$$

$$P(x<3) = \frac{16}{32}$$

(l) Prob of Less than or equal to 2 male children

$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{1}{32} + \frac{5}{32} + \frac{10}{32}$$

$$= \frac{16}{32}$$

(m) ~~at~~ prob of atleast 2 male children

$P(X \geq 2) = ?$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \{ P(X=0) + P(X=1) \}$

$= 1 - \{ \frac{1}{32} + \frac{5}{32} \}$

$= 1 - \frac{6}{32}$

$= \frac{26}{32}$

(n) Prob of atleast 4 male children

$P(X \geq 4) = P(X=4) + P(X=5)$

$= \frac{5}{32} + \frac{1}{32}$

$= \frac{6}{32}$

(o) ~~p~~rob' of atmost 3 male children

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

or $P(X \leq 3) = 1 - P(X > 3)$

$= 1 - [P(X=4) + P(X=5)]$

$= 1 - [\frac{5}{32} + \frac{1}{32}]$

$= 1 - \frac{6}{32}$

$= \frac{26}{32}$

(p) Prob of more than or equal to 3 male children

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \frac{10}{32} + \frac{5}{32} + \frac{1}{32}$

$= \frac{16}{32}$

(q) 2 female children $\Rightarrow$ 3 male children

$$P(X=3) = {}^5C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^{5-3}$$

$$= {}^5C_3 \left(\tfrac{1}{2}\right)^5 = \frac{10}{32}$$

$$P(x=r) = {}^nC_r\, p^r\, q^{n-r}$$

Problem for practice

(2) A coin is tossed 6 times. Find the probability of getting

(1) no heads (2) one head (2) two heads
(3) Three heads (4) 4 heads (5) 5 heads
(6) six heads (7) atleast one head (8) atleast Two heads (9) more than 4 heads
(10) Less Than 3 heads (11) more Than 5 heads
(12) Less Than 2 heads (13) All Tails.

Hint: A coin tossed 6 times. $(n=6)$

p = prob of head $= 1/2$
$q = 1-p = 1/2$ = prob of tail.

(3) output of a production process is known to be 30% defective. What is the probability that a sample of 5 items would contain 0, 1, 2, 3, 4, 5 defectives?

Ans: no. of items in a sample $= 5$

$\therefore n = 5$

30% of the items are defective

prob of defective items $= p = 30\% = 0.3$

$q = 1-p = 1-0.3 = 0.7$

(i) pro of no head $r=0$ $= 1.1 \frac{1}{2\times2\times2\times2\times2\times2}$

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

$P(X=1) = {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0}$

Sol: (i) Prob of '0' defectives

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

$P(x=0) = {}^5C_0 (0.3)^0 (0.7)^{5-0}$

$= 1 \cdot (0.3)^0 (0.7)^5$ - $\boxed{(0.7) \wedge 5}$ = 0.16807

$= 1 \cdot 1 \cdot (0.1681)$

$= 0.1681$

(ii) $P(x=1) = {}^5C_1 (0.3)^1 (0.7)^{5-1}$

$= 5 \cdot (0.3)(0.7)^4$

$= 0.3602$

(iii) $P(x=2) = {}^5C_2 (0.3)^2 (0.7)^{5-2}$

$= (10)(0.3)^2 (0.7)^3$

$= 0.3087$

(iv) $P(x=3) = {}^5C_3 (0.3)^3 (0.7)^{5-3}$

$= {}^5C_3 (0.3)^3 (0.7)^2$

$= 10 (0.3)^3 (0.7)^2$

$= 0.1323$

(v) $P(x=4) = {}^5C_4 (0.3)^4 \cdot (0.7)^{5-4}$

$= 5 (0.3)^4 (0.7)^1$

$= 0.0283$

(vi) $P(x=5) = {}^5C_5 (0.3)^5 (0.7)^{5-5}$

$= 1 (0.3)^5 (0.7)^0$

$= 1 \cdot (0.3)^5 \cdot 1$

$= 0.0024$

(4) NOV 2005

The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates atleast 4 passed the examination.

Sol:

no. of candidates in a group = 6

$n = 6$.

percentage of failure = 40% = 0.40

$\therefore$ prob of pass = 60% = 0.60

$\therefore p = (0.60)$

$q = 1 - p = 1 - 0.6 = 0.4$

In a group of 6 candidates,

the prob that atleast 4 passed the exam

ie $P(X \geq 4)$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$P(X=4) = 6C_4 (0.6)^4 (0.4)^2 = (15)(0.1296)(0.16)$
$= 0.31$

$P(X=5) = 6C_5 (0.6)^5 (0.4)^1 = (6)(0.078) = 0.187$

$P(X=6) = 6C_6 (0.6)^6 (0.4)^0 = (1)(0.047) = 0.047$

$P(X \geq 4) = 0.31 + 0.187 + 0.047$
$= 0.544$

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5) April 2004 | NOV 2003

The probability that a man hits the target is $\boxed{\frac{1}{4}}$. If he fires 7 times what is the probability of his hitting the target atleast twice

Sol: no of attempts $n = 7$

prob of hitting the target $p = \frac{1}{4} = 0.25$

$\therefore q = 1 - p = 1 - 0.25 = 0.75$

Prob of hitting the target atleast twice

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - \{P(X=0) + P(X=1)\}$$

1 − 0, 1

0 1 | 2 3 4 5 6 7

$$P(X=r) = n_{c_r} p^r q^{n-r}$$
$$P(X=0) = 7_{c_0} (0.25)^0 (0.75)^7$$
$$= 1 \cdot 1 \cdot (0.1335)$$
$$= 0.1335$$

It will be easy to solve it by

$$P(X=1) = 7_{c_1} (0.25)^1 (0.75)^{7-1}$$
$$= 7_{c_1} (0.25)^1 (0.75)^6$$
$$= (7)(0.25)(0.178)$$
$$= 0.31$$

$$P(X \geq 2) = 1 - \{P(X=0) + P(X=1)\}$$
$$= 1 - \{0.1335 + 0.31\}$$
$$= 1 - \{0.4435\}$$
$$= 0.56.$$

(6) NOV 2002

In a family of 4 children, find the probability that there are (i) exactly one boy (ii) atleast two boys (iii) all are boys.

Sol: no. of children in a family $= 4$

$n = 4$

Prob for a boy $= p = \frac{1}{2} \Rightarrow q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$

(i) prob that there is exactly one boy.

$P(X=1) = ?$

$P(X=r) = nc_r \, p^r q^{n-r}$

$P(X=1) = 4c_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}$

$= (4)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3$

$= (4)\left(\frac{1}{2}\right)^4$

$= \frac{4}{16} = \frac{1}{4}$

(ii) Prob that there are atleast two boys

$P(X \geq 2) = ?$

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$P(X=2) = 4c_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 6\left(\frac{1}{16}\right) = \frac{6}{16}$

$P(X=3) = 4c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = 4\left(\frac{1}{16}\right) = \frac{4}{16}$

$P(X=4) = 4c_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = 1 \cdot \left(\frac{1}{16}\right) = \frac{1}{16}$

$P(X \geq 2) = \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$

(iii) Prob That all are boys

$\Rightarrow$ all The 4 children are boys

$P(X=4) = {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$

$= 1 \left(\frac{1}{16}\right) \cdot 1$

$= \frac{1}{16}$

7 | Nov 2001 |

If x has a binomial distribution with parameters $n=5$ and $p=1/3$; find $P(X \geq 1)$ and $P(X=3)$

Sol : Given $n=5$

$p = 1/3$

$\therefore q = 1-p = 1-1/3 = 2/3$

$\boxed{P(X \geq 1) = 1 - P(X=0)}$

$P(X=r) = {}^nC_r \, p^r \, q^{n-r}$

$P(X=0) = {}^5C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{5-0}$

$= (1)(1)(2/3)^5$

$= 1 \cdot 1 \cdot (0.67)^5$

$P(X=0) = 0.132$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.132$

$= 0.87$

POISSON DISTRIBUTION

This distribution is applicable when

(i) The probability of Success (p) is very small

(ii) the number of Trials n is very Large ($n \to \infty$)

The situations may be no. of phone calls, no. of printing mistakes in a page ... etc where the number n is not limited ($n \to \infty$)

It is applied in those situations where happenning of an event can be counted, but non occurence of an event cannot be known.

Definition

Let $X = \{x_1, x_2, x_3, \ldots x_n, \ldots\}$ be discrete random variable. If the probability of 'x' success is given by.

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

then the distribution is said to be following a Poisson Distribution.

where λ = Average or Mean occurence of an event

$$\lambda = np$$

Problems on Poisson Distribution

$$\boxed{P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}}$$

① The average number of phone calls during a hour at a reception of hotel is 3. Find the probability that there will be

(i) no phone call

(ii) exactly one phone call

(iii) atleast one phone call

(iv) more than 2 phone calls

(v) Less than 3 phone calls during an hour?

Sol :

Average no. of phone calls per hour is 3

ie $\lambda = 3/\text{hr}$

$e^{-\lambda} = e^{-3} = 0.04979$

(i) Prob of no phone call

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^{0}}{0!} = \frac{(0.04979)(1)}{1} = \frac{0.04979}{1}$$

shift - ln - number

(ii) prob of exactly one phone call

$$P(X=1) = \frac{e^{-\lambda} \cdot \lambda^{1}}{1!} = \frac{e^{-3} (3)^1}{1}$$

$$= \frac{(0.04979)(3)}{1}$$

$$= 0.1494$$

(iii) prob of atleast one phone call

$$P(X \geq 1) = 1 - P(X<1)$$

$$= 1 - P(X=0)$$

$$= 1 - 0.04979$$

$$= 0.9502$$

(iv) prob of more than 2 phone calls.

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-3} \cdot (3)^0}{0!} = \frac{(0.04979) \cdot 1}{1} = 0.04979$$

$$P(X=1) = \frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{e^{-3} \cdot 3^1}{1!} = \frac{(0.04979)(3)}{1} = 0.1494$$

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^{2}}{2!} = \frac{e^{-3} \cdot (3)^2}{2} = \frac{(0.04979)(9)}{2} = \cancel{0.1120}\ 0.2240$$

$P(X>2) = 1 - P(X \le 2)$

$P(X>2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - [0.04979 + 0.1494 + \cancel{0.1120}\ 0.2240]$

$= 1 - \cancel{0.3112}\ 0.4232$

$= \cancel{0.6888}\ 0.5768$

(v) prob of Less Than 3 phone calls

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= 0.04979 + 0.1494 + \cancel{0.1120}\ 0.2240$

$P(X<3) = \cancel{0.3112}\ 0.4232$

(2) April 2005 *****

It is Known that an items produced by a Certain machine will be defective is 0.01. It looks very Sm

In a random sample of 100 items, find the probability that there are

(i) no defective (ii) atleast one defective

(iii) not more Than one defective.

$\lambda = np = 100 \times 0.01$

$= 1$

Sol: Given no. of items $n = 100$

prob of defective $p = 0.01$

$\therefore \lambda = np$ = Average no. of defective items

$= (100)(0.01)$

$\lambda = 1$

$e^{-\lambda} = e^{-1} = 0.3679$

(i). Prob of no defective items

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-1}(1)^0}{1} = \frac{(0.3679)(1)}{1} = 0.3679$$

(ii) prob of atleast one defective

$$P(X \geq 1) = 1 - P(X<1)$$
$$= 1 - P(X=0)$$
$$= 1 - 0.3679$$
$$= 0.6321$$

(iii) not more than one defective

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-1} \cdot (1)^0}{0!} = \frac{(0.3679) \cdot 1}{1} = 0.3679$$

$$P(X=1) = \frac{e^{-\lambda} \lambda^1}{1!} = \frac{e^{-1} \cdot (1)^1}{1} = \frac{(0.3679) \cdot 1}{1} = 0.3679$$

$$P(X \leq 1) = 0.3679 + 0.3679 = 0.7358$$

3. Between 2 pm to 4 pm the average no. of phone calls during a particular ~~minute~~ minute at a retaurant is 2.5. Find the prob of getting (i) no phone call (ii) exactly two phone calls (iii) atleast one phone call (iv) more than 3 phone calls during a particular minute.

Sol: average no. of phone calls per minute

$$\lambda = 2.5/\text{min}$$

$$e^{-\lambda} = e^{-2.5} = 0.08208$$

(i) prob of no phone call

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{e^{-2.5} (2.5)^0}{0!}$$

$$= \frac{(0.08208) \cdot 1}{1} = 0.08208$$

(ii) prob of exactly two phone calls

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^2}{2!} = \frac{e^{-2.5} \cdot (2.5)^2}{2!}$$

$$= \frac{(0.08208)(6.25)}{2}$$

$$= 0.2565$$

(iii) Prob of atleast one phone call

$$P(X \geq 1) = 1 - P(X<1)$$
$$= 1 - P(X=0)$$
$$= 1 - 0.08208$$
$$= 0.918$$

(iv) ∴ Prob of more than 3 phone calls

$$P(X>3) = 1 - P(X \leq 3)$$
$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$P(X=0) = 0.08208$$

$$P(X=1) = \frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{e^{-2.5}(2.5)}{1} = (0.08208)(2.5) = 0.2052$$

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^2}{2!} = \frac{e^{-2.5} \cdot (2.5)^2}{2!} = 0.2565$$

$$P(X=3) = \frac{e^{-\lambda} \cdot \lambda^3}{3!} = \frac{e^{-2.5}(2.5)^3}{6} = 0.214$$

$$P(X>3) = 1 - [0.08208 + 0.2052 + 0.2565 + 0.214]$$
$$= 1 - 0.758$$
$$= 0.242$$

(4) If 5% of the electric bulbs manufactured by a Company are defective, use poisson distribution to find the probability that in a sample of 100 bulbs

(i) none is defective

(ii) 5 bulbs will be defective

(given $e^{-5} = 0.007$)

Sol: no. of bulbs $n = 100$

prob of defective $p = 5\% = \frac{5}{100} = 0.05$

$\therefore \lambda = np = 100 \times 0.05$

$\lambda = 5$

$e^{-\lambda} = 0.007$

(i) prob that none is defective

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \frac{e^{-5} (5)^{0}}{0!} = \frac{(0.007) \cdot 1}{1}$$

$$= 0.007$$

(ii) Prob of 5 defective bulbs

$$P(x=5) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \frac{e^{-5} (5)^{5}}{5!} = \frac{(0.007)(3125)}{120}$$

$$= 0.1823$$

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(5) A car hire firms has two cars which it hires out day by day. The number of demands for car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used; (ii) Some demand is rejected.

Sol:

$$\lambda = 1.5 \text{ / day}$$

$$e^{-\lambda} = e^{-1.5} = 0.2231$$

(I) Prob of neither car is used

$$P(X=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{(0.2231)(1.5)^0}{1} = 0.2231$$

(ii) Prob that some demand is rejected

$$P(X>2) = 1 - P(X \le 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=0) = 0.2231$$

$$P(X=1) = \frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{e^{-1.5}(1.5)^1}{1!} = \frac{(0.2231)(1.5)}{1} = 0.3346$$

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^2}{2!} = \frac{e^{-1.5}(1.5)^2}{2!} = \frac{(0.2231)(1.5)^2}{2!} = 0.251$$

$$P(X>2) = 1 - [0.2231 + 0.3346 + 0.251]$$

$$= 1 - 0.8087 = 0.1913$$

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